

S.Y.Novak

Extreme value methods with applications to finance.

Monographs on Statistics and Applied Probability, 122.

CRC Press, Boca Raton, FL, 2012. xxvi+373 pp. ISBN: 978-1-4398-3574-6

AMS Subject Classification:60E15, 60F05, 60G50, 60G55, 60G70, 62G05, 62G32, 62P05, 91G70, 91G80, 97M30.

Extreme value theory (EVT) is a rapidly growing branch of modern probability and statistics. It has important applications in actuarial and financial mathematics, hydrology, meteorology, and other fields.

The classical EVT that deals with independent random variables has been fully developed by the 1970s and is well presented in several books.

EVT for dependent random variables encounters problems which were not present in the classical setup. This has inspired the development of EVT for stationary sequences of random variables.

The book presents modern topics in EVT such as

- (*) Poisson approximation to the number of rare events,
- (*) compound Poisson approximation,
- (*) extremes in samples of random size,
- (*) empirical point processes of exceedances,
- (*) Poisson cluster approximation,
- (*) nonparametric estimation methods,
- (*) methods of estimating extreme quantiles and tail probabilities,
- (*) nonparametric lower bounds,
- (*) self-normalized sums of random variables and Student's statistic,
- (*) measures of financial risk.

The novelty of this book in comparison to other texts on EVT topics is the detailed coverage of a wide range of topics. The author is an expert in EVT, and many his own results are included in the book.

The book is divided into two parts:

Part I, "Distribution of Extremes",

Part II, "Statistics of Extremes".

Probabilistic aspects of EVT are given in Part I, which consists of eight chapters. Methods of EVT (Bernstein's "blocks" method, the "runs" approach, and the method of recurrent inequalities) are presented in Chapter 1.

The Erdős-Rényi maximum of partial sums (MPS) is studied in Chapter 2. MPS is a universal statistic that covers a wide range of statistics from sums to maxima, and links the asymptotic theory of sums of random variables (TSRV) and EVT.

Asymptotic properties of extreme values in samples of random size are investigated in Chapter 3. The important particular case is when the sample size is a renewal process. The problem is related to that of the length of the longest head-run (LLHR) and to the problem of the length of the longest match pattern (LLMP), which have been the topics of considerable interest; the problems have applications in actuarial mathematics and statistical analysis of DNA data.

The next three chapters deal with the number N_n of exceedances of a "high" level. If the random variables are independent, then N_n has the binomial distribution and can be well

approximated by a Poisson random variable. New results on the accuracy of Poisson are presented in Chapter 4.

Dependent random variables are considered in Chapters 4 and 5. The only possible limiting distribution of N_n in typical situations is compound Poisson. Sharp estimates of the accuracy of compound Poisson approximation to the distribution of N_n are presented in Chapter 5.

Chapter 6 deals with compound Poisson approximation to the distribution of a vector of rare events. The topic has insurance and reinsurance applications, where N_n stands for the number of claims exceeding certain levels.

Results concerning the one-dimensional empirical point processes of exceedances that count either locations of extremes or their heights are presented in Chapter 7.

A general two-dimensional empirical point process of exceedances N_n^* that counts both locations of extremes and their heights is studied in Chapter 8. The class of possible limiting distributions for N_n^* has been described, and necessary and sufficient conditions for the weak convergence (“complete convergence”) of N_n^* to a limiting two-dimensional point process have been presented. The feature of the result is that the limiting two-dimensional process is a “composition” of two one-dimensional processes, one of which is pure Poisson.

The statistical part of the book consists of six chapters.

It is well known that financial and insurance data often exhibit heavy tails. The main parameter describing the heavy tail is the so-called tail index (exponent of regular variation). The theory of statistical inference on heavy tails from a sample of dependent random variables is developed in Chapter 9. The chapter presents accurate nonparametric estimators of the tail index, extreme quantiles, tail probabilities and second-order indices.

Chapter 10 is devoted to modern approaches to financial risk measurement. Among popular measures of risk are Value-at-Risk (VaR) and conditional VaR (CVaR), which is also known as Expected Shortfall (ES). Accurate methods of VAR and ES estimation are presented. The chapter introduces also a dynamic measure of risk.

The notion of extremal index in relation to the distribution of extremes is studied in Chapter 11. Necessary and sufficient conditions for the stationary sequence to possess an extremal index are presented. Estimators of the extremal index are suggested using the so-called “runs” and “blocks” approaches. The asymptotic properties of these estimators are established.

Chapter 12 presents sharp estimates of the accuracy of normal approximation to the distribution of self-normalized sums (SNS) of random variables and Student’s statistic.

One section of chapter 12 is devoted to the Stein method. The author presents new results on characterisation of a wide class of distributions via the Stein method.

Nonparametric lower bounds with explicit constants to the accuracy of tail index and extreme quantiles estimation are derived in Chapter 13.

An appendix, Chapter 14, is of interest on its own. It contains many useful auxiliary results, e.g., probability distributions and their properties, probabilistic identities, distances, large deviations, elements of renewal theory, dependence, point processes, slowly varying functions, identities and inequalities, including a probabilistic version of Taylor’s formula and identities concerning interplay between binomial and Poisson distributions.

Theoretical results are illustrated by examples and applications to particular problems of financial risk measurement.

A large number of exercises and open problems are displayed all over the text.

The bibliography of 407 items is an excellent source of references on the topics presented in the book.

<https://www.crcpress.com/Extreme-Value-Methods-with-Applications-to-Finance/Novak/9781439835746>

<https://www.routledge.com/products/9781439835746> <http://www.crcnetbase.com/isbn/978-1-4398-3574-6>